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MATHEMATICS

1 CHAPTER Commercial Mathematics

Goods and Services Tax

- Selling Price = List Price – Discount
- Tax = Rate of tax × SP.
- Intra state (Same state)
  - CGST
  - SGST
- Inter State (Different State)
  - IGST
- Input tax Credit (Business purpose only)
  - Output tax – Input tax
  - Tax on SP – Tax on CP (Tax on profit)

Banking

- Recurring Deposit account (R.D. Account)
  - Interest = $\boxed{P \times \frac{n(n+1)}{2 \times 12} \times \frac{r}{100}}$
  - Maturity Value = $\boxed{P \times n + \text{Interest}}$

Shares & Dividends

- The original value of a share is called NOMINAL VALUE.
- The price of a share at any time is called its MARKET VALUE.
- If the market valve of a share is the same as its NOMINAL VALUE, the share is called at PAR.
- If the market value of a share is more than its NOMINAL VALUE, then the share is called at PREMIUM or ABOVE PAR.
- If the market value of a share is less than its NOMINAL VALUE, then the share is called at DISCOUNT or BELOW PAR.

$$\boxed{\begin{array}{c}\text{Sum Invested} = \text{Number of shares} \times \text{Market value of share} \\ \Rightarrow \text{No. of shares} = \dfrac{\text{Sum Invested}}{\text{Market Value of Share}}\end{array}}$$

$$\boxed{\begin{array}{c}\Rightarrow \text{No. of shares} = \dfrac{\text{Total Dividend}}{\text{Dividend of 1 share}} \\ \Rightarrow \dfrac{\text{Total Income (Profit)}}{\text{Income (Profit) on 1 share}}\end{array}}$$

$$\boxed{\text{Total Dividend} = \text{No. of share} \times \text{Rate of dividend} \times \text{N.V. of share}}$$

$$\boxed{\text{Rate of dividend} \times \text{N.V.} = \text{Profit (return)\%} \times \text{M.V.}}$$

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2 CHAPTER Linear Inequations

Linear Inequations

Signs of inequality are $>$, $<$, $\geq$ and $\leq$.

RULE 1: Positive term transferred to other side become negative.

Ex.: $3x + 2 < 8 \Rightarrow 3x < 8 - 2$

RULE 2: Negative term transferred to other side become positive.

Ex.: $x - 3 > 5 \Rightarrow x > 5 + 3$

RULE 3: If each term is multiplied or divided by same positive number, sign of inequality remains same.

Ex.: $3x > 6 \Rightarrow \frac{3x}{3} > \frac{6}{3} \Rightarrow x > 2$ divided by 3

$\frac{x}{2} < 5 \Rightarrow 2 \times \frac{x}{2} < 5 \times 2 \Rightarrow x < 10$ multiplied by 2

RULE 4: Taking reciprocal on both sides reverses inequality sign.

Ex.: $x > 3 \Rightarrow \frac{1}{x} < \frac{1}{3}$

RULE 5: If each term is multiplied or divided by same negative number, sign of inequality reverses.

Ex.: $-2x < 5 \Rightarrow \frac{-2x}{-2} < \frac{5}{-2} \Rightarrow x > \frac{-5}{2}$ divided by -2

RULE 6: If sign on both side changes, inequality reverses.

Ex.: $x < -2$, $-x > 2$

Replacement & Solution Set

- If $x > 4$

  Replacement set = $x \in N$ (x belongs to natural number)

  Solution set = {1, 2, 3}

- Number line – ○ Hollow represent $>$, $<$

  ● Filled represents $\geq$, $\leq$.

3 CHAPTER Quadratic Equations

Quadratic Equation

An equation with one variable, in which the highest power of variable is two, is known as quadratic equation.

Standard form: $ax^2 + bx + c$, where a, b & c are all real numbers and $a \neq 0$.

- Every quadratic equation gives two values of the unknown variable used in it & these valves are called ROOTS/SOLUTION of the quadratic equation.
- For the quadratic equation $ax^2 + bx + c = 0, a \neq 0$; the expression $b^2 - 4ac$ is called DISCRIMINANT & is denoted by D.
- The roots of the quadratic equation $ax^2 + bx + c = 0$; where $a \neq 0$ can be obtained by

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Also expressed as $\boxed{x = \frac{-b \pm \sqrt{D}}{2a}}$, where $D = b^2 - 4ac$

Nature of Roots

Nature of roots of a quadratic equation depends on DISCRIMINANT.

$\boxed{D = b^2 - 4ac}$

- $D > 0$: The roots are real and unequal.
- $D = 0$: The roots are real and equal.
- $D < 0$: No real root exists.

4
CHAPTER

Ratio and Proportion

Ratio

If a and b are two quantities of the same kind and with the same units such that $b \neq 0$; then the quotient $\frac{a}{b}$ is called the ratio between a and b.

Commensurable and incommensurable quantities

If the ratio between any two quantities of the same kind and having the same unit can be expressed exactly by the ratio between two integers; the quantities are said to be commensurable; otherwise incommensurable.

Composition of ratios:

(i) Compound Ratio

For two or more ratios, the ratio between the product of their antecedents to the product of their consequents is called compound ratio.

e.g. for ratios $a : b$ and $c : d$; the compound ratio is $(a \times c) : (b \times d)$.

(ii) Duplicate ratio

It is the compound ratio of two equal ratios.

e.g. Duplicate ratio of $a : b$ = compound ratio of $a : b$ and $a : b$

$= (a \times a) : (b \times b) = a^2 : b^2$

(iii) Triplicate ratio

It is the compound ratio of three equal ratios.

e.g. Triplicate ratio of $a : b$ = Compound ratio of $a : b$, $a : b$ and $a : b$

$= (a \times a \times a) : (b \times b \times b) = a^3 : b^3$

(iv) Sub-duplicate ratio

For any ratio $a : b$, its sub-duplicate ratio is $\sqrt{a} : \sqrt{b}$

(v) Sub-triplicate ratio

For any ratio $a : b$ its sub-triplicate ratio is $\sqrt[3]{a} : \sqrt[3]{b}$

(vi) Reciprocal ratio

For any ratio $a : b$; where $a, b \neq 0$, its reciprocal ratio $= \frac{1}{a} : \frac{1}{b} = b : a$.

Proportion

Four non-zero quantities a, b, c and d are said to be proportion, if a : b :: c : d and is read as "a is to b as c is to d".

Some Properties of Ratios & Proportion

- **Invertendo:** If a : b :: c : d then b : a : d : c = $\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c}$

- **Alternendo:** If a : b :: c : d then a : c :: b : d = $\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a}{c}=\frac{b}{d}$

- **Componendo:** If a : b :: c : d then (a + b) : b :: (c + d) : d = $\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a}{b}+1=\frac{c}{d}+1 \Rightarrow \frac{a+b}{b}=\frac{c+d}{d}$

- **Dividendo:** If a : b :: c : d then (a – b) : b :: (c – d) : d = $\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{a}{b}-1=\frac{c}{d}-1 \Rightarrow \frac{a-b}{b}=\frac{c-d}{d}$

- **Componendo & Dividendo:** If a : b :: c : d then (a + b) : (a – b) :: (c + d) : (c – d)

  On dividing the results of Componendo by Divendo, we get = $\frac{a+b}{b} \times \frac{b}{a-b}=\frac{c+d}{d} \times \frac{d}{c-d} \Rightarrow \frac{a+b}{a-b}=\frac{c+d}{c-d}$

- **Convertendo:** If a : b :: c : d then a : (a – b) :: c : (c – d) $\frac{a}{b}=\frac{c}{d}$ ....(i)

  By dividendo, $\frac{a-b}{b}=\frac{c-d}{d}$ ...(ii)

  Diving (i) by (ii), we get, $\frac{a}{b} \times \frac{b}{a-b}=\frac{c}{d} \times \frac{d}{c-d} \Rightarrow \frac{a}{a-b}=\frac{c}{c-d}$

- If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then each ratio = $\frac{a+c+e}{b+d+f}$

5
CHAPTER

Remainder and Factor Theorems

Remainder Theorem

When a polynomial $f(x)$ (whose degree is greater than or equal to 1) is divided by a linear polynomial $(x - a)$, the remainder is given by $R = f(a)$.

Factor Theorem

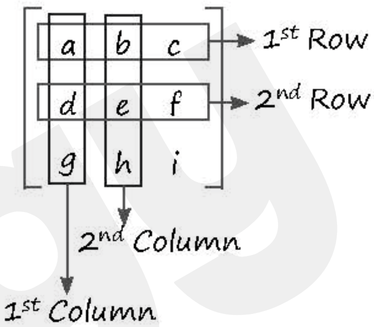
When a polynomial $f(x)$ is divided by $x - a$, the remainder $= f(a)$. And if $f(a) = 0$;
$\Rightarrow (x - a)$ is a factor of the polynomial $f(x)$.

6
CHAPTER

Matrices

Matrices

A rectangular arrangement of numbers in the form of horizontal & vertical lines is called a matrix.



- Horizontal lines are called ROWS & Vertical lines are COLUMNS.
- If a matrix contains m rows & n column then it is called a matrix of order mxn.
- A matrix of order mxn has mn elements.
- An element appearing in the i^{th} row & j^{th} column of a matrix is called its $(i, j)^{th}$ element or the $(i, j)^{th}$ entry.

Equal Matrices

Two matrices A & B are called equl, written as A = B, if and only if

(i) A and B are of same order i.e. No. of rows in A = No. of rows in B & No. of columns in A = No. of columns in B.

(ii) their corresponding elements are equal i.e. the entries of A & B in the same position are equal.

Special Types of Matrices

Row Matrix:

A matrix having only one row is called row matrix.

Column Matrix:

A matrix having only one column is called column matrix.

Square Matrix:

A matrix in which number of rows equals the number of columns is called Square Matrix.

Zero or Null Matrix:

A matrix whose each element is zero is called a zero or null matrix.

Identity or Unit Matrix:

A square matrix in which each diagonal element is 1 and all other elements are zero is called an identity or unit matrix.

Addition of Matrices

If A & B are two matrices of the same order, we say that these are compatible for addition, and their sum A + B is the matrix obtained by adding the corresponding elements of A & B.

Subtraction of Matrices

If A & B are two matrices of the same order, we say that these are compatible for subtraction, and their difference A − B is the matrix obtained by subtracting the corresponding elements of B from A.

Transpose of a Matrix

Transpose of a matrix is the matrix obtained on interchanging its rows & columns. If A is matrix then its transpose is denoted by A^t.

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 4 & 7 \end{bmatrix}, \text{ then } A^t = \begin{bmatrix} 2 & 0 \\ 3 & 4 \\ 1 & 7 \end{bmatrix}$$

Additive Identity

If any number is added to zero or zero is added to the number, the number remains the same. 0 is said to be **additive identity** in numbers.

If any matrix is added to null matrix of same order then the matrix remains the same. So, here **null matrix** is said to be **additive identity** in matrices.

Additive Inverse

If A & B are two matrices of the same order such that:

A + B = B + A = a null matrix; then A is said to be **additive inverse** of B & B is said to be **additive inverse** of A.

Multiplication of Matrices

Two matrices A & B can be multiplied together to get the product matrix AB. If & only if, the numbers of columns in A (the left hand matrix) is equal to the number of rows in B (the right hand matrix).

$$A_{m \times n} \times B_{n \times p} = C_{m \times p} = AB$$

Ex: $A = \begin{bmatrix} 3 & 4 \\ 5 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\quad AB = \begin{bmatrix} (3\times1)+(4\times3) & (3\times2)+(4\times4) \\ (5\times1)+(0\times3) & (5\times2)+(0\times4) \end{bmatrix}$

Identity matrix for multiplication:

On multiplying any matrix with the identity matrix of the same order, the matrix remains unaltered.

$$A \times I = A = I \times A$$

Points to remember:

1. For any three matrices A,B, and C which are compatible for multiplication; ABC = A(BC). Similarly, BAC = B(AC); CAB = C(AB) and so on.
2. In general $AB \neq BA$ i.e. product of matrices is not commutative.
3. (AB) C = A(BC) i.e. product of matrices is associative.
4. If $A \neq 0$ and AB = AC. then it is not necessary that B = C.
5. If AB = 0, then it is not necessary that A = 0 or B = 0.
6. If A = 0 or B = 0, then AB = 0 = AB.
7. (i) A(B + C) = AB + AC i.e. in matrices, multiplication is distributive over addition.
   (ii) (A+B) C = AC + BC.
   In the same way, A (B − C) = AB − AC and (A − B) C = AC − BC.

7 CHAPTER Arithmetic and Geometric Progression

Arithmetic & Geometric Progressions

A list of number is called an ARITHMETIC PROGRESSION if and only if the difference of any term from its preceding term is constant.

- An A.P. is called a finite A.P. if and only if it contains finite number of terms.
- An A.P. is called an infite A.P. if and only if it contains infinite number of terms.
- Let $a_1, a_2, a_3........l$ is an A.P. then
  $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = d$, where d = common difference and ℓ is last term.
- $\boxed{n^{th} \text{ term of an A.P.} = a + (n - 1)d}$, where a = first term & d = common difference
- $\boxed{n^{th} \text{ term of an A.P. from last} = \ell + (n - 1)(-d)}$

Sum of first n terms of an A.P. = $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$

or $\boxed{S_n = \frac{n}{2}[a + l]}$

- Three terms in A.P. = $(a - d), a, (a + d)$
- Four terms in A.P. = $(a - 3d), (a - d), (a + d), (a + 3d)$; where common difference = 2d.
- Five terms in A.P. = $(a - 2d), (a - d), a, (a + d), (a + 2d)$
- Let say, $S_n = n^2 - n$

  $\Rightarrow S_1 = a_1$

  $\Rightarrow S_2 - S_1 = a_2$

  $\Rightarrow a_2 - a_1 = d$

Geometric Progression

A list of non-zero numbers is called a geometric progression (G.P.) if and only if the ratio of its any term to its preceding term is constant. i.e., fixed number $a_1, a_2, a_3,.........$is a G.P. if and only if $\boxed{\frac{a_{n+1}}{a_n} = r}$, where r is constant & independent of n.

General term of a Geometric Progression

Let a be the first term and L be last term and r $(r \neq 0)$ be the common ratio of G.P. Then n^{th} form of GP is $\boxed{a_n = ar^{n-1}}$ and k^{th} term from end of GP (finds GP) $T_k = ar^{n-k}$ where T_k is k^{th} term from the end, and n is total number terms.

Geometric Mean

If a & b be two positive real numbers, then $\sqrt{ab}$ is called Geometric mean between a & b.

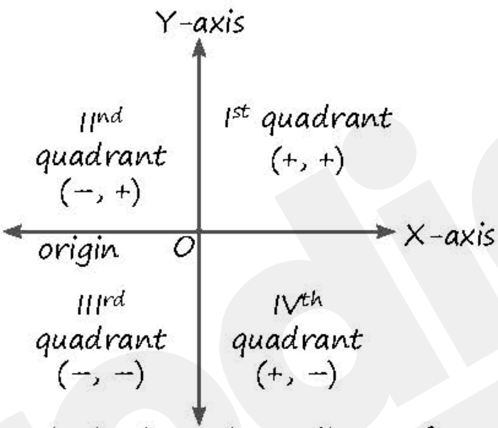
Sum of a geometric progression

Sum of G.P.

- If $r = 1$: $S_n = n \times a$
- If $|0<|r|<|$: $S_n = \dfrac{a(1-r^n)}{1-r}$
- If $|r|>1$: $S_n = \dfrac{a(r^n-1)}{r-1}$

8 CHAPTER Co-ordinate Geometry

Coordinate Geometry



Abscissa: The abscissa represents the horizontal coordinate of a point, typically denoted as X in a Cartesian coordinate system.

Ordinate: The ordinate represents the vertical coordinate of point, usually denoted as y in a Cartesian coordinate system.

Coordinates of Origin $\rightarrow (0, 0)$
Coordinates of a point on X-axis $\rightarrow (X, 0)$
Coordinates of a point on Y-axis $\rightarrow (0, Y)$

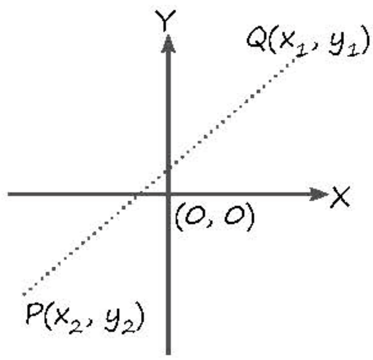
- General equation of line, $y = mx + c$

  Equation of line passing through origin $y = mx$

  Equation of x-axis $\rightarrow Y = 0$

  Equation of y-axis $\rightarrow X = 0$

Distance Formula



$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Distance of a point (x, y) from origin $= \sqrt{x^2 + y^2}$

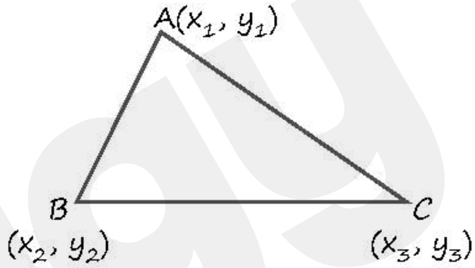
Section Formula

Internal $= \left[\dfrac{m_1x_2+m_2x_1}{m_1+m_2}, \dfrac{m_1y_2+m_2y_1}{m_1+m_2}\right]$

External $= \left[\dfrac{m_1x_2-m_2x_1}{m_1-m_2}, \dfrac{m_1y_2-m_2y_1}{m_1-m_2}\right]$

Mid-point formula $= \left[\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right]$

Area of Triangle

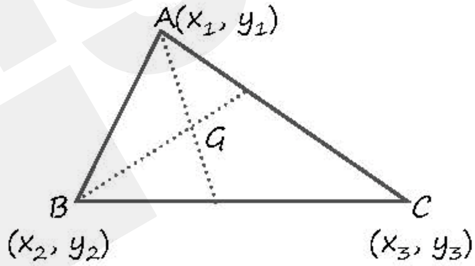


Area of $\Delta = \dfrac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

If points $(x_1 y_1)$, (x_2, y_2), (x_3, y_3) are collinear, then

Area of $\Delta = 0$

Centroid of Triangle



$$\boxed{G = \frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}}$$

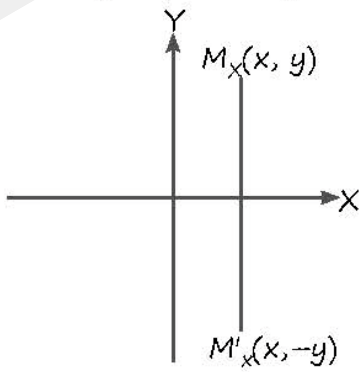
- Centroid divides the median into 2: 1.

Reflection

When an object is placed before a plane mirror, the image formed is at the same distance behind the mirror as the object is in front of it.

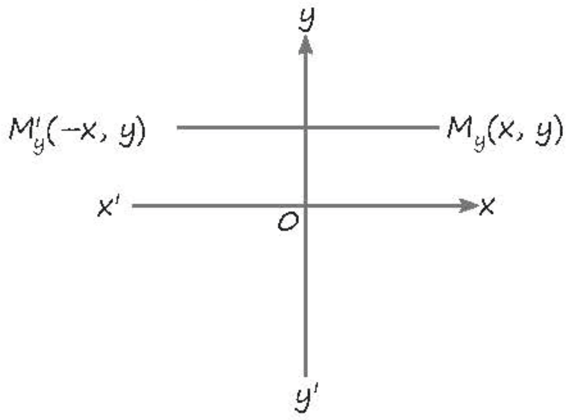
- Reflection in the line Y = 0, i.e., x-axis

$M_x(x, y) = (x, -y)$



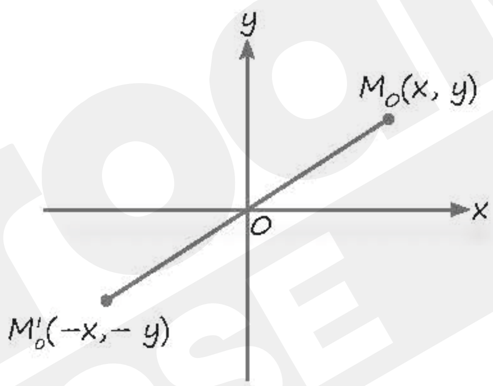
- Reflection w.r.t. line X = 0, i.e., Y-axis

$$M_y(x, y) = (-x, y)$$



❑ Reflection w.r.t. Origin

$$M_o(x, y) = (-x, -y)$$



9 CHAPTER Similarity

Similarity

Figures which have exactly the same shape and the same size are said to be congruent.

- Figures which have exactly the same shape, but not necessarily the same size are said to be similar.
- All line segments are similar.
- All squares are similar.
- All equilateral triangles are similar.
- All circles are similar.

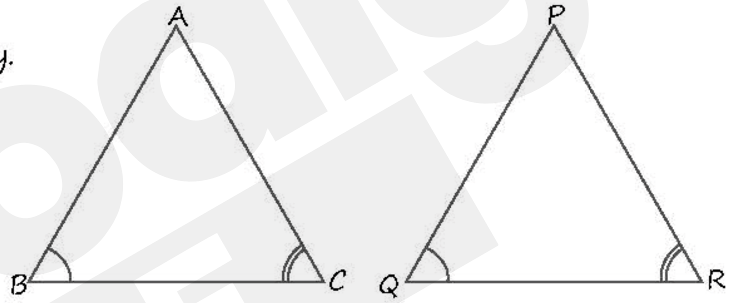
Rules For Similarity of Triangles

- AA (Angles-Angle) rule for similarity.

$\angle B = \angle Q$

$\angle C = \angle R$

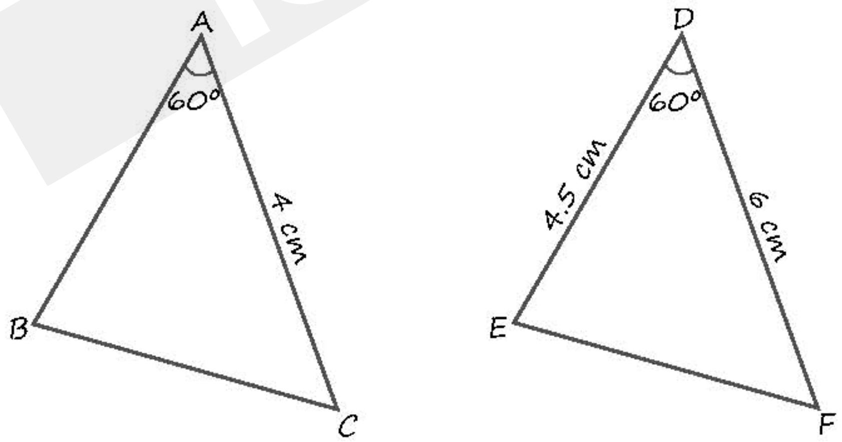
$$\boxed{\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}}$$



$\Delta ABC \sim \Delta PQR$

If two angles of a triangle are equal to two angles of another triangle, then the two triangles are similar.

- SAS (Side-Angle-Side) rule of similarity $\frac{AB}{DE} = \frac{AC}{DF} = \frac{2}{3}$ & $\angle A = \angle D$

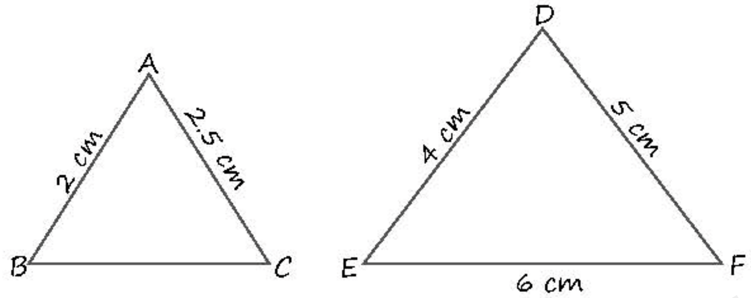


$\Delta ABC \sim \Delta DEF$

If one angle of triangle is equal to one angle of another angle & the sides including the angle are proportional, then two triangles are similar.

- SSS (side-side-side) rule of similarity.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$$



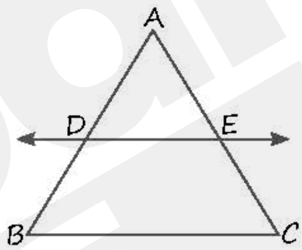
- If tow triangle have their three pairs of corresponding sides proportional, then the triangle are similar.

Basic Proportional Theorem (Thales Theorem)

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divides in the same ratio.

If $DE \parallel BC$

$$\frac{AD}{DB} = \frac{AE}{EC}$$

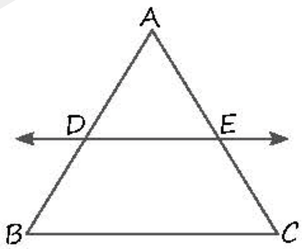


Converse of Basic Proportionality Theorem

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

If $\frac{AD}{DB} = \frac{AE}{EC}$

Therefore, $DE \parallel BC$



Relation between the areas of two similar triangles:

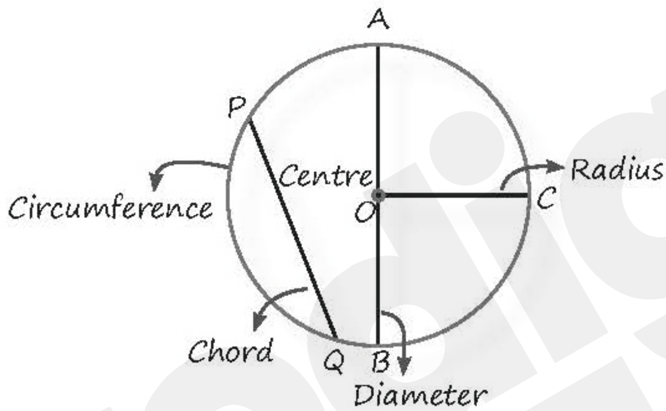
The areas of two similar triangle are proportional to the squares on their corresponding sides.

If $\Delta ABC \sim \Delta DEF$, then

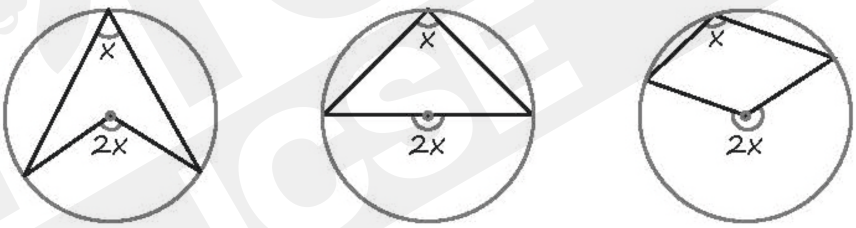
$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

10 CHAPTER Circles

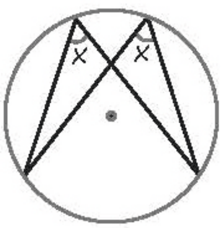
Circles



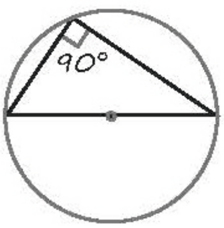
- Angle at centre and segment



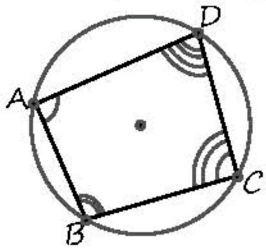
- Angle in same segment → Equal



- Angle in semicircle → 90°

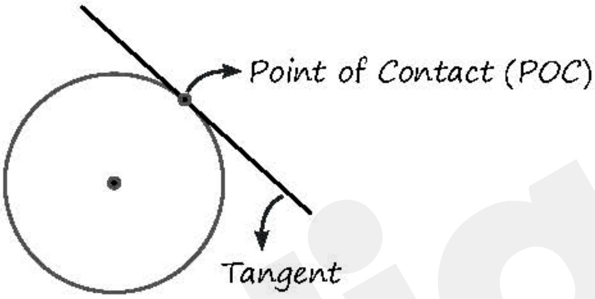


- Cyclic Quadrilateral → Sum of opposite angles of a cyclic quadrilateral is 80°

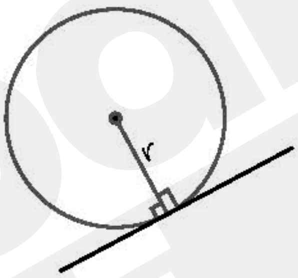


$\angle A + \angle C = 180°$
$\angle B + \angle D = 180°$

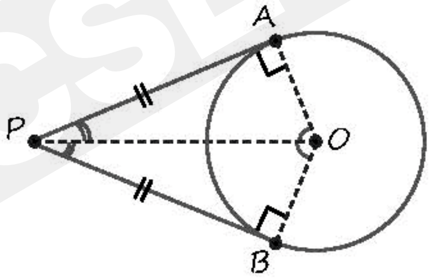
- Tangent → Line that touch circle at only one point.



- Radius is Perpendicular to POC of tangent.

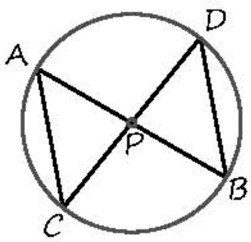


- Tangent from external point.



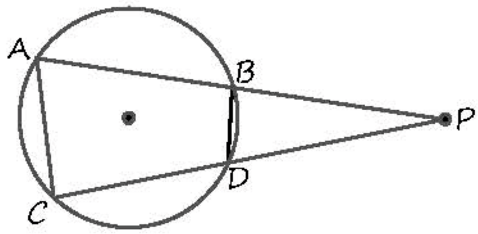
$\Rightarrow PA = PB$
$\Rightarrow \angle APO = \angle BPO$
$\Rightarrow \angle POA = \angle POB$

- Intersection of Chords
  - Internally



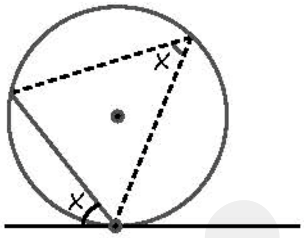
$\boxed{PA \times PB = PC \times PD}$

+ Externally

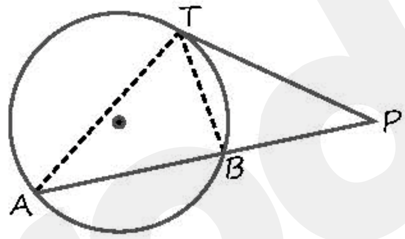


$\boxed{PA \times PB = PC \times PD}$

- Alternate segment theorem



- Tangent - Secant Theorem



$\boxed{PA \times PB = PT^2}$

11 CHAPTER Mensuration (Surface Area and Volume)

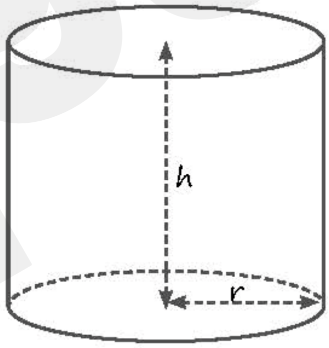
Surface Area and Volume

- The sum of the areas of all the surfaces of a solid is called its SURFACE AREA or its TOTAL SURFACE AREA.
- The surface area of a solid excluding the areas of its top & bottom is called LATERAL or curved SURFACE AREA.
- The space occupied by a solid is called its VOLUME.
- If a solid is cut perpendicular to its length (breadth or height), the plane surface so obtained is called its CROSS-SECTION.

Cylinder

A solid which has uniform cross-section is called a cylinder. Let r be the radius & h be the height.

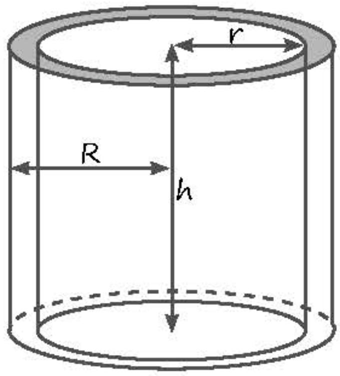
- Area of cross-section = πr^2
- Perimeter of cross-section = $2\pi r$
- Curved (Lateral) Surface Area = $2\pi r.h$
- Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
- Volume = $\pi r^2 h$



Hollow Cylinder

Let R be the external radius & r be the internal radius & h be its height.

- Thickness of the wall = $R - r$
- Area of cross-section = $\pi(R^2 - r^2)$
- External curved surface area = $2\pi Rh$
- Internal curved surface area = $2\pi rh$
- Total surface area = $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$
- Volume of material = $\pi(R^2 - r^2).h$

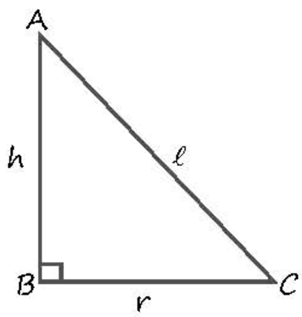


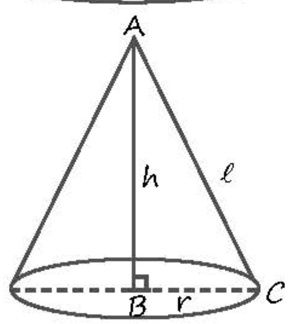
Cone

The solid obtained on revolving a right angled triangle about one of its sides (other than hypotenuse). is called a right circular cone.

In ΔABC, $l^2 = h^2 + r^2$

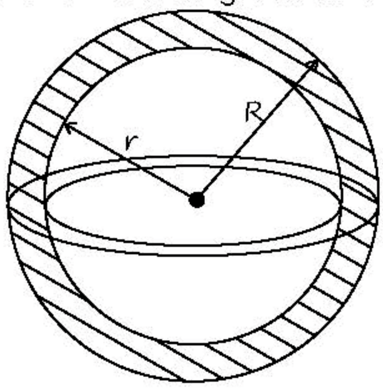
- Volume = $\frac{1}{3}\pi r^2 h$
- Curved or lateral surface area = πrl
- Total surface area = $\pi rl + \pi r^2 = \pi r(l + r)$





Sphere

A sphere is a solid obtained on revolving a circle about any diameter of it.



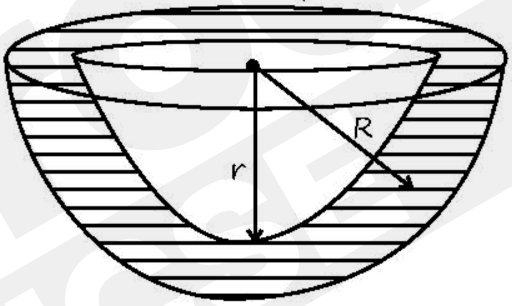
- Volume of sphere = $\frac{4}{3}\pi R^3$ = Volume of material in sphere
- Surface Area of Sphere $=4\pi r^2$

Spherical Shell

It is a solid enclosed between two spheres.

Thickness of shell = $R - r$

- Volume = $\frac{4}{3}\pi\left(R^3 - r^3\right)$ = Volume of material in spherical shell.



Hemisphere

- Volume of hemisphere = $\frac{2}{3}\pi R^3$
  Curved surface area = $2\pi r^2$
- Total surface area = $3\pi R^2$

Hemispherical shell

- Thickness of shell = $R - r$
- Area of bass = $\pi(R^2 - r^2)$
- External curved surface area = $2\pi R^2$
- Internal curved surface area = $2\pi r^2$
- Total surface area = $2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$
- Volume of material = $\frac{2}{3}\pi\left(R^3 - r^3\right)$

Note

- The total volume of the solids to be concerted is equal to the total the volume of the solid into which the given solid is to be concerted. although, the surface area may change.

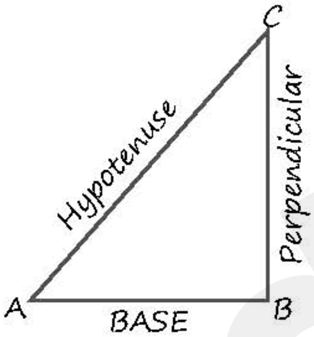
12 CHAPTER Trigonometry

Trigonometry

$\sin A = \dfrac{BC}{AC}$ $\quad$ $\operatorname{cosec} A = \dfrac{AC}{BC}$

$\cos A = \dfrac{AB}{AC}$ $\quad$ $\sec A = \dfrac{AC}{AB}$

$\tan A = \dfrac{BC}{AB}$ $\quad$ $\cot A = \dfrac{AB}{BC}$



$$\sin A = \frac{1}{\operatorname{cosec} A} = \frac{\text{Perpendicular}}{\text{Hypotenuse}}$$

$$\cos A = \frac{1}{\sec A} = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\tan A = \frac{1}{\cot A} = \frac{\sin A}{\cos A} = \frac{\text{Perpendicular}}{\text{Base}}$$

Trigonometric Identities

$\sin^2 A + \cos^2 A = 1, \quad 0^\circ \le A \le 90^\circ$

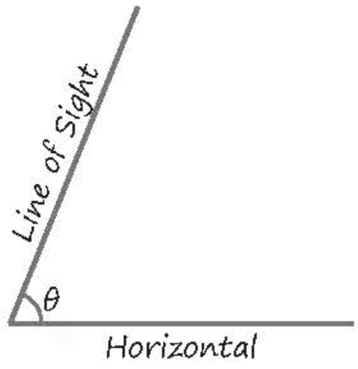
$\sec^2 A - \tan^2 A = 1, \quad 0^\circ \le A \le 90^\circ$

$\cos^2 A - \cot^2 A = 1, \quad 0^\circ \le A \le 90^\circ$

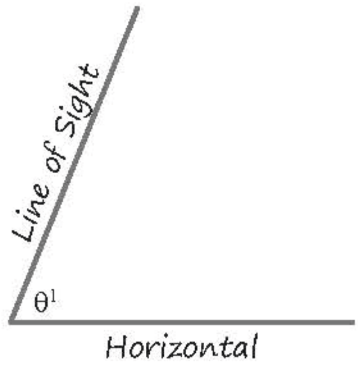
Trigonometric Ratio Table

	0°	45°	60°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined
$\operatorname{cosec}\theta$	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined
$\cot\theta$	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

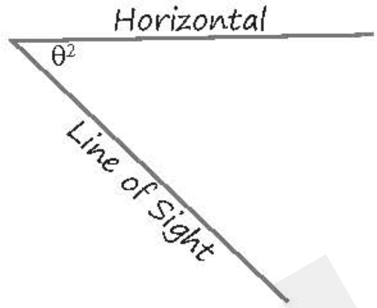
Heights & Distance



Angle of Elevation



Angle of Depression



13 CHAPTER Statistics

Statistics

- Mean = $\frac{\text{Sum of observation}}{\text{No. of observation}}$

- Class mark $\boxed{x_i = \frac{\text{Upper limit} + \text{lower limit}}{2}}$

  + Direct Method

Classes	f_i	x_i	fx_i

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

  + Short-Cut Method

Classes	f_i	x_i	$4_i = \frac{x_i - A}{h}$	$f_i d_i$
		$\square$ A		

$\bar{x} = \frac{\Sigma f_i U_i}{\Sigma fi} + h$ where A = Assumed Mean.

  + Step Deviation Method

Classes	f_i	x_i	$4_i = \frac{x_i - A}{h}$	$u_1 + di/n$	$f_i U_i$
		$\square$ A			

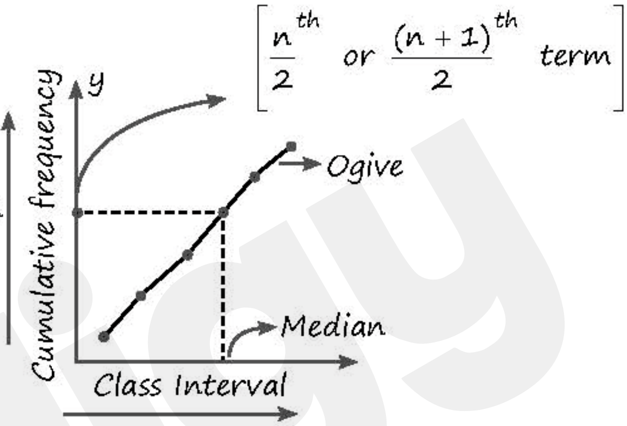
$\bar{x} = A + \frac{\Sigma f_i U_i}{\Sigma fi} \times h$ where h = class size

- Median – Middle Value of data arranged in ascending or descending order. If these are 'n' observations, then

$$\text{Median} = \begin{cases} \left(\dfrac{n+1}{2}\right)^{th} \text{observation, if } n \text{ is odd} \\[2ex] \dfrac{\left(\dfrac{n}{2}\right)^{th} \text{observation} + \left(\dfrac{n}{2}+1\right)^{th} \text{observation}}{2}, \text{ if } n \text{ is even} \end{cases}$$

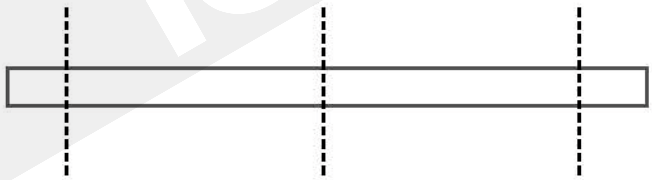
+ Tabulated Data – Make a column for cumulative frequency
  n = total no. of observations
  Depending on n is even or odd find the median

+ Grouped Data –



1. Make CF column
2. Draw a cumulative frequency curve (ogive)
3. It there be n terms in the ogive distribution then by the help of given find $\left(\dfrac{n}{2}\right)^{th}$ or $\left(\dfrac{n+1}{2}\right)^{th}$ term which will be the median of the given distribution.

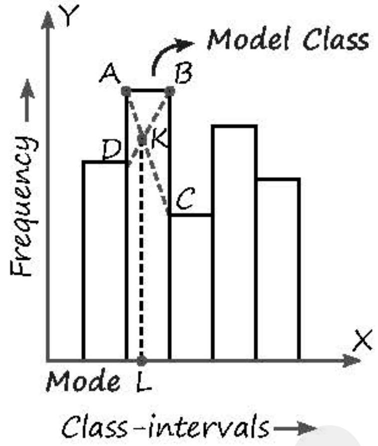
❑ Quartiles



Lower Quartile (Q_1)		Median (Q_2)	Upper Quartile (Q_3)	
$\left(\dfrac{n}{4}\right)^{th}$ term	$\left(\dfrac{n+1}{4}\right)^{th}$ term		$\left(\dfrac{3n}{4}\right)^{th}$ term	$\left(\dfrac{3(n+1)}{4}\right)^{th}$ term
n = (even)	n = (odd)		n = (even)	n = (odd)

+ Inter Quartile range = $Q_3 - Q_1$, which is always positive as $Q_3 > Q_1$.

- Mode – The value with maximum frequently in a given data set.
  - Grouped Data – Histogram



1. Draw a histogram of the given distribution.
2. Inside the highest rectangle, which is of maximum frequency (Modal class), Draw two lines AC and BD diagonals from the upper corners C and D of adjacent rectangles.
3. Let K be the point of intersection of diagonals AC and BD.
4. Draw KL perpendicular to horizontal axis.
5. The value of point L on the horizontal axis represents the value of mode.